2.9 PBLAS routines for matrix-vector operations (Level 2)

Originally, L3 BLAS specification is carried out for the FORTRAN language and the defined/included subroutines are:

a. "General" matrix products (subroutines ending in GEMM):

$C \neg a\ op(A)\ op(B) + bC$

where $op(X)$ can be X, X^T o X^H

b. Matrix products where one of the matrices is real or symmetrical complex or hermitical complex (subroutines ending in SYMM or HEMM):

$C \neg aAB + bC$ *or* $C \neg aBA + bC$

where A is symmetrical for SYMM or hermitical for HEMM and is located to the left or right of the multiplication depending on a subroutine parameter (SIDE).

c. Matrices products where one of them is triangular (subroutines ending in TRMM):

$B\neg\ a\ op(A)\ B$ *or* $B\neg\ a\ B\ op(A)$

where A is a triangular matrix; it is to the left or right of the multiplication depending on

a subroutine parameter (SIDE), and $op(A)$ can be A, A^To A^H.

d. Rank-k update of a symmetrical matrix (subroutines ending in SYRK):

$C \neg aAA^T + bC$ *or* $C \neg aA^TA + bC$

where C is symmetrical and A is to the right or left of the multiplication by A^T, depending on a subroutine parameter (TRANS).

e. Rank-k update of a hermitical matrix (subroutines ending in HERK):

$C \neg aAA^H + bC$ *or* $C \neg aA^HA + bC$

where C is hermitical and A is to the right or left of the multiplication by A^H, depending on a subroutine parameter (TRANS).

f. Rank-2k update of a symmetrical matrix (subroutines ending in SYR2K):

$C \neg aAB^T + aBA^T + bC$ *or* $C \neg aA^TB + aB^TA + bC$

where C is symmetrical and A is to the right or left of the multiplication by B^T, depending on a subroutine parameter (TRANS).

g. Rank-2k update of a hermitical matrix (subroutines ending in HER2K):

$C \neg aAB^H + alfa\ BA^H + bC$ *or* $C \neg aA^HB + alfa\ B^HA + bC$

where C is hermitical and A is to the right or left of the multiplication by B^H, depending on a subroutine parameter (TRANS).

h. Solutions to triangular equations systems (subroutines ending in TRSM):

$B\neg\ a\ op(A)\ B$ *or* $B\neg\ a\ B\ op(A)$

where A is a triangular matrix; it is to the left or right of the multiplication depending on a subroutine parameter (SIDE), and $op(A)$ can be A^{-1}, A^{-T}o A^{-H}.

Matrix multiplication has very specific characteristics as regards the design and implementation of a parallel algorithm in the parallel algorithms context in general:

- Computation independency: each element computed from the result matrix C, *cij*, is, in principle, independent of all the other elements. This independence is utterly useful because it allows a wide flexibility degree in terms of parallelization.
- Data independence: the number and type of operations to be carried out are independent of the data. In this case, the exception is the algorithms of the so-called sparse matrix multiplication, where there exists an attempt to take advantage of the fact that most of the matrices elements to be multiplied (and thus, of the result matrix) are equal to zero.
- Regularity of data organization and of the operations carried out on data: data are organized in two-dimensional structures (the same matrices), and the operations basically consist of multiplication and addition.

Cannon's Algorithm.

It is also proposed for a two-dimensional array of processing elements interconnected as a mesh and with the edges of each row and column interconnected, i.e. making up a structure called torus for *3´3* processing elements.

Initially, matrices *A, B,* and *C* data distribution is similar to that defined previously in the mesh, i.e. if the processors are numbered according to their position in the two-dimensional array, processor P_{ij} $(0 \le i, j \le P-1)$ has the elements or blocks of position ij $(0 \le i, j \le P-1)$ of matrices *A, B* and *C*. In order to simplify the explanation, matrices elements shall be used instead of blocks. From this data distribution, matrices *A* and *B* data are "realigned" or reassigned so that, if there is a two-dimensional array of $P \times P$ processors, the element or submatrix *A* in row *i* and column $(j+i) \bmod P$, $a_{i,(j+i) \bmod P}$, and also the element or submatrix of *B* in row $(i+j) \bmod P$ and column *j*, $b_{(i+j) \bmod P,i}$, are assigned to processor P_{ij}. In other words, each data of row i $(0 \le i \le P-1)$ of the elements or submatrices of *A* are transferred or shifted *i* times towards the left processors, and each data or column j $(0 \le j \le P-1)$ of the elements or submatrices of *B* are transferred or shifted j times towards upper processors.

From the initial relocation, the following steps are carried out iteratively:

- Local multiplication of data assigned in each processor for a partial result computation;
- Left rotation of the elements or submatrices of *A*;
- Upwards rotation of the elements or submatrices of *B*, and after *P* of these steps, thoroughly computed values of matrix *C* are finally obtained.

Summarizing, the outstanding characteristics of this way to carry out matrix multiplication are:

- By the way matrices *A* and *B* data are communicated, this is an "initial alignment and rotating" algorithm.
- Load balance, both in terms of computation and communications, is assured only if the processing algorithms are homogeneous.
- As in the processing defined for the processors grid, the running time is minimized if the computation can be overlapped in time with communications.
- Matrices *A* and *B* data distribution is not the initial one when the matrix multiplication computation is finished.

Finally, the **Cannon's Algorithm** consists of the folowing steps:

1) The **initial step** of the algorithm regards the alignment of the matrixes:

a) – Align the blocks of A and B in such a way that each process can independently start multiplying its local submatrices. This is done by shifting all submatrices $A_{i,j}$ to the left (with wraparound) by **i** steps a and all submatrices $B_{i,j}$ up (with wraparound) by **j** steps.

2) Perform local block multiplication.

3) Each block of *A* moves **one step left** and each block of *B* moves **one step up** (again with wraparound).
4) Perform next block multiplication, add to partial result, repeat until all blocks have been multiplied.

Cannon's Algorithm for 3×3 Matrices and process (1,2) to determine C_{12}

A(0, 0_	A(0, 1)	A(0, 2)
A(1, 0)	A(1, 1)	A(1, 2)
A(2, 0)	A(2, 1)	A(2, 2)

B(0, 0_	B(0, 1)	B(0, 2)
B(1, 0)	B(1, 1)	B(1, 2)
B(2, 0)	B(2, 1)	B(2, 2)

Initial A, B

A(0, 0_	A(0, 1)	A(0, 2)
A(1, 1)	A(1, 2)	A(1, 0)
A(2, 0)	A(2, 1)	A(2, 2)

B(0, 0_	B(0, 1)	B(2, 2)
B(1, 0)	B(1, 1)	B(0, 2)
B(2, 0)	B(2, 1)	B(1, 2)

A, B initial alignment

A(0, 0_	A(0, 1)	A(0, 2)
A(1, 2)	A(1, 0)	A(1, 1)
A(2, 0)	A(2, 1)	A(2, 2)

B(0, 0_	B(0, 1)	B(0, 2)
B(1, 0)	B(1, 1)	B(1, 2)
B(2, 0)	B(2, 1)	B(2, 2)

A, B after shift step 1

A(0, 0_	A(0, 1)	A(0, 2)
A(1, 0)	A(1, 1)	A(1, 2)
A(2, 0)	A(2, 1)	A(2, 2)

B(0, 0_	B(0, 1)	B(1, 2)
B(1, 0)	B(1, 1)	B(2, 2)
B(2, 0)	B(2, 1)	B(0, 2)

A, B after shift step 2

So $C(1,2)=A(1,0)*B(0,2)+A(1,1)*B(1,2)+ A(1,2)*B(2,2)$ and according the Cannon's Algorithm we have:

- $C^1(1,2)= A(1,0)*B(0,2)$ (after A, B initial alignment)
- $C^2(1,2)= C^1(1,2)+A(1,1)*B(1,2)$ (A, B after shift step 1)
- $C(1,2)=C^3(1,2)= C^2(1,2)+A(1,2)*B(2,2)$ (A, B after shift step 2)

Atension! Limitations of Cannon's Algorithm

- P (number of processors) is must be a perfect square;
- Matrices A and B must be square, and evenly divisible by √p

Scalable Universal Matrix Multiply Algorithm (SUMMA)

O altenativa a Cannon algoritm este SUMMA Algorithm:
SUMMA = Scalable Universal Matrix Multiply

- Slightly less efficient, but simpler and easier to generalize
- Presentation from van de Geijn and Watts
- www.netlib.org/lapack/lawns/lawn96.ps
- Similar ideas appeared many times
- Used in practice in PBLAS = Parallel BLAS

- www.netlib.org/lapack/lawns/lawn100.ps

Naive matrix multiply

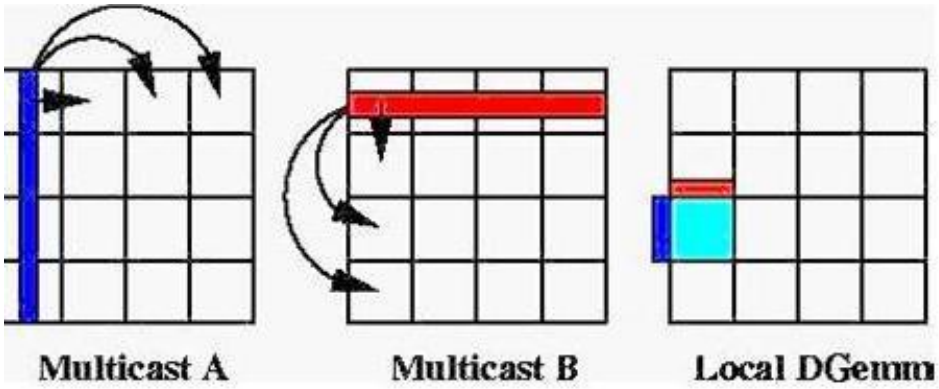
- *For i = 0 to n*
- *For j = 0 to n*
- *For k = 0 to n*
- *C[i,j] += A[i,k]*B[k,j]*

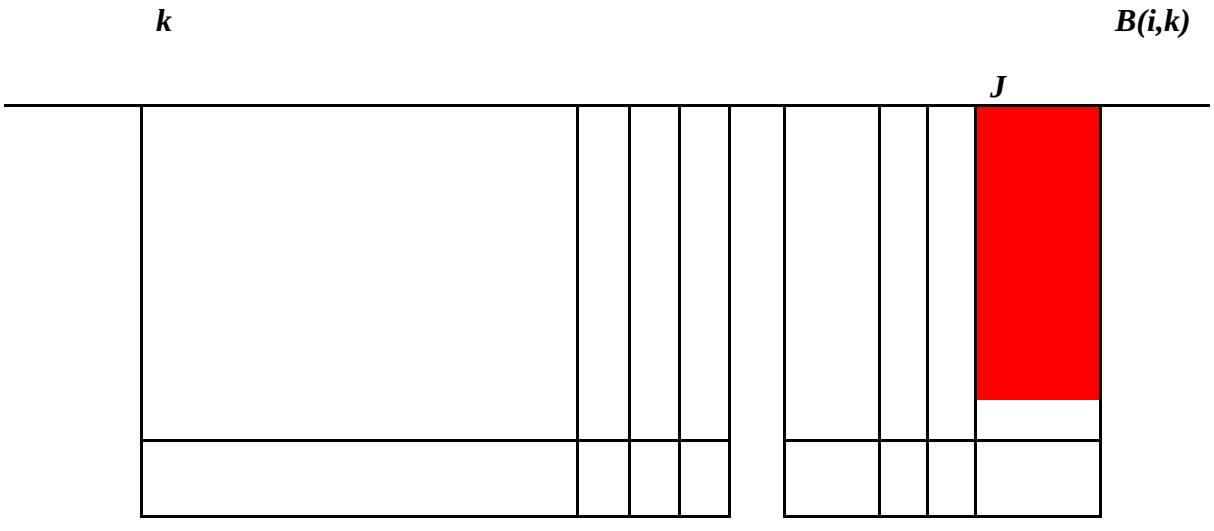
Calculates n^2 dot products (inner products) C[i,j] = A[i,:]*B[:,j]

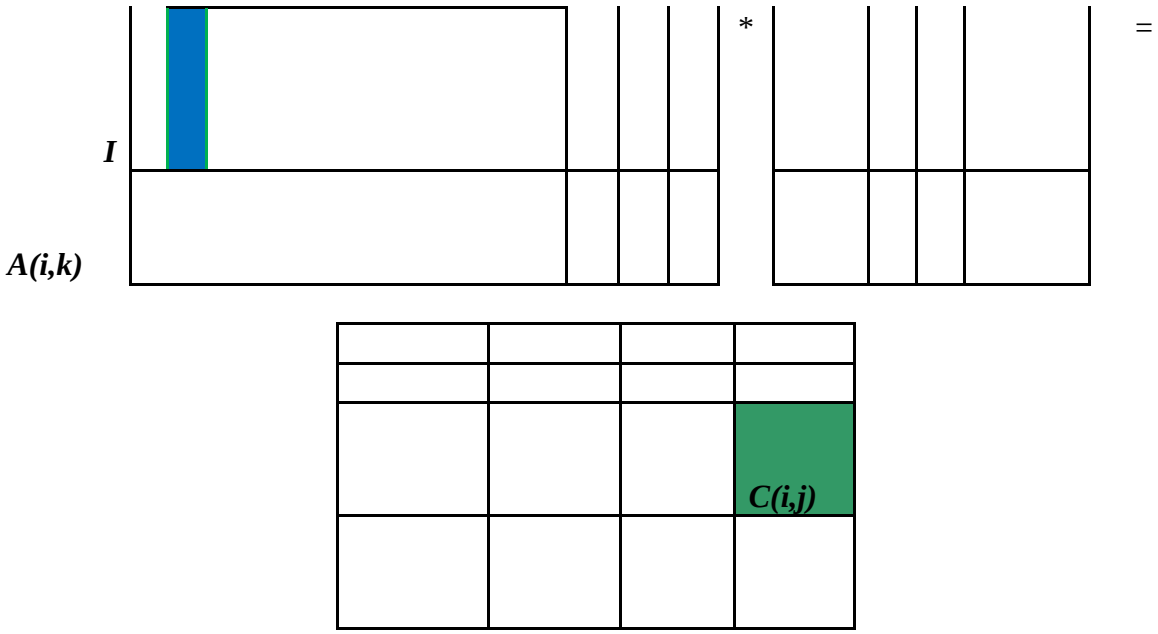
The main steps of the algorithm:

a) For each *k* (between 0 and n-1),

- Owner of partial row *k* broadcasts that row along its process column;
- Owner of partial column *k* broadcasts that column along its process row







b) I, J represent all rows, columns owned by a processor

c) k is a single row or column (or a block of ***b*** rows or columns)

d) C(I,J) = C(I,J) +$\sum_k$ A(I,k)*B(k,J)

Assume a p_r by p_c processor grid ($p_r = p_c = 4$ above)

Complete algorithm. On each process P(i,j):

- **For** *k=0* to *n-1* … or *n/b-1* where b is the block size
  …= # cols in A(I,k) and # rows in B(k,J)

- for all $I = 1$ to p_r … in parallel
- owner of *A(I,k)* broadcasts it to whole processor row
- for all J = 1 to pc … in parallel
- owner of *B(k,J)* broadcasts it to whole processor column
- Receive A(I,k) into Acol
- Receive B(k,J) into Brow
- C(myproc , myproc) = C(myproc , myproc) + Acol * Brow

♦ **Endfor**

```
PvGEMV(TRANS,M,N,ALPHA,A,IA,JA,DESCA,X,IX,JX,DESCX,INCX,
BETA,Y,IY,JY,DESCY,INCY)
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```
void pdgemv_(char *trans, int *m, int *n, double
*alpha, double *A, int *ia, int *ja, int *desc_A,
double *X, int *ix, int *jx,  int*desc_X, int *incx,
```

```
        double *beta, double *Y, int *iy, int *jy, int *desc_Y,
        int *incy );
```

Purpose PvGEMV performs one of the distributed matrix-vector operations

*sub(Y) := alpha*sub(A)´sub(X) + beta×sub(Y)*, or *sub(Y) := alpha×sub(A)^T × sub(X) + beta×sub(Y),*

where *sub(A) denotes A(IA:IA+M-1,JA:JA+N-1),*

$$\text{sub}(X) = \begin{cases} \begin{cases} X(IX:IX, JX:JX+N-1) & \text{if INCX} = \text{M\_X} \\ X(IX:IX+N-1, JX:JX) & \text{if INCX} = 1 \text{ and INCX} <> \text{M\_X} \end{cases} & \text{if TRANS} = \text{'N'} \\ \begin{cases} X(IX:IX, JX:JX+M-1) & \text{if INCX} = \text{M\_X} \\ X(IX:IX+M-1, JX:JX) & \text{if INCX} = 1 \text{ and INCX} <> \text{M\_X} \end{cases} & \text{else} \end{cases}$$

$$\text{sub}(Y) = \begin{cases} \begin{cases} Y(IY:IY, JY:JY+M-1) & \text{if INCY} = \text{M\_Y} \\ Y(IY:IY+M-1, JY:JY) & \text{if INCY} = 1 \text{ and INCY} <> \text{M\_X} \end{cases} & \text{if TRANS} = \text{'N'} \\ \begin{cases} Y(IY:IY, JY:JY+N-1) & \text{if INCY} = \text{M\_Y} \\ Y(IY:IY+N-1, JY:JY) & \text{if INCY} = 1 \text{ and INCXY} <> \text{M\_Y} \end{cases} & \text{else} \end{cases}$$

alpha and *beta* are scalars, and *sub(X)* and *sub(Y*) are distributed vectors and *sub(A)* is a *M*-by-*N* distributed submatrix.

Arguments

TRANS (global input) CHARACTER
On entry, TRANS specifies the operation to be performed as follows:

$$\begin{cases} \text{sub}(Y) := \text{alpha} * \text{sub}(A) * \text{sub}(X) + \text{beta} * \text{sub}(Y), & \text{if TRANS} = \text{'N'} \\ \text{sub}(Y) := \text{alpha} * \text{sub}(A)' * \text{sub}(X) + \text{beta} * \text{sub}(Y), & \text{if TRANS} = \text{'T'} \\ \text{sub}(Y) := \text{alpha} * \text{sub}(A)' * \text{sub}(X) + \text{beta} * \text{sub}(Y). & \text{if TRANS} = \text{'C'} \end{cases}$$

M (global input) INTEGER

The number of rows to be operated on i.e. the number of rows of the distributed submatrix sub(A). M≥ 0.

N (global input) INTEGER

The number of columns to be operated on i.e the number of columns of the distributed submatrix sub(A). N≥ 0.

ALPHA (global input) REAL/COMPLEX

On entry, ALPHA specifies the scalar alpha.

A (local input) array of dimension (LLD_A,LOCq(JA+N-1))

This array contains the local pieces of the distributed matrix sub(A).

IA (global input) INTEGER

The global row index of the submatrix of the distributed matrix A to operate on.

JA (global input) INTEGER

The global column index of the submatrix of the distributed matrix A to operate on.

DESCA (global and local input) INTEGER array of dimension 8

The array descriptor of the distributed matrix A.

X (local input/local output) array of dimension at least

$$\begin{cases} ((JX-1)*M\_X+IX+(N-1)*\mathrm{abs}(INCX)) & \text{if TRANS}='N' \\ ((JX-1)*M\_X+IX+(M-1)*\mathrm{abs}(INCX)) & \text{else.} \end{cases}$$

This array contains the entries of the distributed vector sub(X).

IX (global input) INTEGER

The global row index of the submatrix of the distributed matrix X to operate on.

JX (global input) INTEGER

The global column index of the submatrix of the distributed matrix X to operate on.

DESCX (global and local input) INTEGER array of dimension 8

The array descriptor of the distributed matrix X.

INCX (global input) INTEGER

The global increment for the elements of X. Only two values of INCX are supported in this version, namely 1 and M_X.

BETA (global input) REAL/COMPLEX

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then sub(Y) need not be set on input.

Y (local input/local output) array of dimension at least

$$\begin{cases} ((JY-1)*M\_Y+IY+(M-1)*\mathrm{abs}(INCY)) & \text{if TRANS}='N' \\ ((JY-1)*M\_Y+IY+(N-1)*\mathrm{abs}(INCY)) & \text{else} \end{cases}$$

This array contains the entries of the distributed vector sub(Y). On exit, sub(Y) is overwritten by the updated distributed vector sub(Y).

IY (global input) INTEGER

The global row index of the submatrix of the distributed matrix Y to operate on.

JY (global input) INTEGER

The global column index of the submatrix of the distributed matrix Y to operate on.

DESCY (global and local input) INTEGER array of dimension 8

The array descriptor of the distributed matrix Y.

INCY (global input) INTEGER
The global increment for the elements of Y. Only two values of INCY are supported in this version, namely 1 and M_Y. \

Notes and Coding Rules

- These subroutines accept lowercase letters for the transa argument.
- For PDGEMV, if you specify 'C' for transa, it is interpreted as though you specified 'T'.
- The matrix and vectors must have no common elements; otherwise, results are unpredictable.
- The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above.
- The following values must be equal: CTXT_A = CTXT_X = CTXT_Y.
- The following coding rules depend upon the values specified for transa and incx:
    - 1. If transa = 'N' and incx = M_X:
        1. The following block sizes must be equal: NB_A = NB_X.
        2. In the process grid, the process column containing the first column of the submatrix *X* must also contain the first column of the submatrix *A*; that is, iacol = ixcol, where:
            1. iacol = mod((((ja-1)/NB_A)+CSRC_A), q)
            2. ixcol = mod((((jx-1)/NB_X)+CSRC_X), q)
        3. The block column offset of *x* must be equal to the block column offset of *A*; that is, mod(jx-1, NB_X) = mod(ja-1, NB_A).

    2. If transa = 'N' and incx = 1(≠ M_X):
        1. The following block sizes must be equal: NB_A = MB_X.
        2. The block row offset of *x* must be equal to the block column offset of *A*; that is, mod(ix-1, MB_X) = mod(ja-1, NB_A).

    3. If transa = 'T' or 'C' and incx = M_X:
        1. The following block sizes must be equal: MB_A = NB_X.
        2. The block column offset of *x* must be equal to the block row offset of *A*; that is, mod(jx-1, NB_X) = mod(ia-1, MB_A).

    4. If transa = 'T' or 'C' and incx = 1(≠ M_X):

1. The following block sizes must be equal:
   MB_A = MB_X.

2. In the process grid, the process row containing the first row of the submatrix *X* must also contain the first row of the submatrix *A*; that is, iarow = ixrow, where:

    1. iarow = mod((((ia-1)/MB_A)+RSRC_A), p)

    2. ixrow = mod((((ix-1)/MB_X)+RSRC_X), p)

3. The block row offset of *x* must be equal to the block row offset of *A*; that is, mod(ix-1, MB_X) = mod(ia-1, MB_A).

- The following coding rules depend upon the values specified for transa and incy:

1. If transa = 'N' and incy = M_Y:
1. The following block sizes must be equal: MB_A = NB_Y.
2. The block column offset of *y* must be equal to the block row offset of *A*; that is, mod(jy-1, NB_Y) = mod(ia-1, MB_A).
2. If transa = 'N' and incy = 1(≠ M_Y):
1. The following block sizes must be equal: MB_A = MB_Y.
2. In the process grid, the process row containing the first row of the submatrix *Y* must also contain the first row of the submatrix *A*; that is, iarow = iyrow, where:
1. iarow = mod((((ia-1)/MB_A)+RSRC_A), p)
2. iyrow = mod((((iy-1)/MB_Y)+RSRC_Y), p)
3. The block row offset of *y* must be equal to the block row offset of *A*; that is, mod(iy-1, MB_Y) = mod(ia-1, MB_A).
3. If transa = 'T' or 'C' and incy = M_Y:
1. The following block sizes must be equal: NB_A = NB_Y.
2. In the process grid, the process column containing the first column of the submatrix *Y* must also contain the first column of the submatrix *A*; that is, iacol = iycol, where:
1. iacol = mod((((ja-1)/NB_A)+CSRC_A), q)
2. iycol = mod((((jy-1)/NB_Y)+CSRC_Y), q)
3. The block column offset of *y* must be equal to the block column offset of *A*; that is, mod(jy-1, NB_Y) = mod(ja-1, NB_A).
4. If transa = 'T' or 'C' and incy = 1(≠ M_Y):
1. The following block sizes must be equal: NB_A = MB_Y.
2. The block row offset of *y* must be equal to the block column offset of *A*; that is, mod(iy-1, MB_Y) = mod(ja-1, NB_A).

PvTRMV(UPLO,TRANS,DIAG,N,A,IA,JA,DESCA,X,IX,JX,DESCX,INCX)

Purpose: PvTRMV performs one of the distributed matrix-vector operations

sub(X) := sub(A) × sub(X) or *sub(X) := sub(A)^T ×sub(X),*

where *sub(A) denotes A(IA:IA+N-1,JA:JA+N-1),*

$$sub(X) = \begin{Bmatrix} X(\mathrm{IX},\mathrm{JX}:\mathrm{JX}+\mathrm{N}-1) & \text{if INCX} = \mathrm{M\_X}, \\ X(\mathrm{IX}:\mathrm{IX}+\mathrm{N}-1,\mathrm{JX}) & \text{if INCX} = 1 \text{ and INCX} <> \mathrm{M\_X}, \end{Bmatrix}$$

sub(X) is an *N* element vector and *sub(A)* is an *N*-by-*N* unit, or non-unit, upper or lower triangular distributed matrix.

Arguments

UPLO (global input) CHARACTER

On entry, UPLO specifies whether the upper or lower triangular part of the distributed matrix sub(A) is to be referenced.

TRANS (global input) CHARACTER

On entry, TRANS specifies the operation to be performed as follows:

TRANS = 'N' sub(x) := sub(A)*sub(x).

TRANS = 'T' sub(x) := sub(A)'*sub(x).

TRANS = 'C' sub(x) := sub(A)'*sub(x).

DIAG (global input) CHARACTER

On entry, DIAG specifies whether or not sub(A) is unit triangular.

N (global input) INTEGER

The order of the distributed matrix sub(A). N≥0.

A (local input) array of dimension (LLD_A,LOC_q(JA+N-1))

This array contains the local pieces of the distributed matrix sub(A). Before entry with UPLO = 'U', the leading N-by-N upper triangular part of the distributed matrix sub(A) must contain the upper triangular distributed matrix and the strictly lower triangular part of sub(A) is not referenced. Before entry with UPLO = 'L', the leading N-by-N lower triangular part of the distributed matrix sub(A) must contain the lower triangular distributed matrix and the strictly upper triangular part of sub(A) is not referenced. Note that when DIAG = 'U', the diagonal elements of sub(A) are not referenced either, but are assumed to be unity.

IA (global input) INTEGER

The global row index of the submatrix of the distributed matrix A to operate on.

JA (global input) INTEGER

The global column index of the submatrix of the distributed matrix A to operate on.

DESCA (global and local input) INTEGER array of dimension 8

The array descriptor of the distributed matrix A.

X (local input/local output) array of dimension at least ((JX-1)*M_X+IX+(N-1)*abs(INCX))

This array contains the entries of the distributed vector sub(X).

IX (global input) INTEGER

The global row index of the submatrix of the distributed matrix X to operate on.

JX (global input) INTEGER

The global column index of the submatrix of the distributed matrix X to operate on.

DESCX (global and local input) INTEGER array of dimension 8

The array descriptor of the distributed matrix X.

INCX (global input) INTEGER
The global increment for the elements of X. Only two values of INCX are supported in this version, namely 1 and M_X.

PvTRSV(UPLO,TRANS,DIAG,N,A,IA,JA,DESCA,X,IX,JX,DESCX,INCX)
void pdtrsv_(F_CHAR_T UPLO, F_CHAR_T TRANS, F_CHAR_T DIAG,
int *N, double *A, int *IA, int *JA, int *DESCA,
double *X, int *IX, int *JX, int *DESCX, int *INCX)

Purpose: PvTRSV solves one of the systems of equations

sub(A) ´ sub(X) = b, or $sub(A)^{T}$´ *sub(X) = b,*

where *sub(A)* denotes *A(IA:IA+N-1,JA:JA+N-1),*

$$sub(X) = \begin{cases} X(\mathrm{IX}, \mathrm{JX} : \mathrm{JX} + \mathrm{N} - 1) & \text{if INCX} = \mathrm{M\_X}, \\ X(\mathrm{IX} : \mathrm{IX} + \mathrm{N} - 1, \mathrm{JX}) & \text{if INCX} = 1 \text{ and INCX} <> \mathrm{M\_X}, \end{cases}$$

Arguments

UPLO (global input) CHARACTER
On entry, UPLO specifies whether the upper or lower triangular part of the distributed matrix sub(A) is to be referenced.
TRANS (global input) CHARACTER
On entry, TRANS specifies the operation to be performed as follows:
TRANS = 'N' sub(x) := sub(A)*sub(x).
TRANS = 'T' sub(x) := sub(A)'*sub(x).
TRANS = 'C' sub(x) := sub(A)'*sub(x).
DIAG (global input) CHARACTER
On entry, DIAG specifies whether or not sub(A) is unit triangular.
N (global input) INTEGER
The order of the distributed matrix sub(A). N≥0.
A (local input) array of dimension (LLD_A,LOC_q(JA+N-1))
This array contains the local pieces of the distributed matrix sub(A). Before entry with UPLO = 'U', the leading N-by-N upper triangular part of the distributed matrix sub(A) must contain the upper triangular distributed matrix and the strictly lower triangular part of sub(A) is not referenced. Before entry with UPLO = 'L', the leading N-by-N lower triangular part of the distributed matrix sub(A) must contain the lower triangular distributed matrix and the strictly upper triangular part of sub(A) is not referenced. Note that when DIAG = 'U', the diagonal elements of sub(A) are not referenced either, but are assumed to be unity.
IA (global input) INTEGER
The global row index of the submatrix of the distributed matrix A to operate on.
JA (global input) INTEGER
The global column index of the submatrix of the distributed matrix A to operate on.
DESCA (global and local input) INTEGER array of dimension 8
The array descriptor of the distributed matrix A.

X (local input/local output) array of dimension at least ((JX-1)*M_X+IX+(N-1)*abs(INCX))
This array contains the entries of the distributed vector sub(X).
IX (global input) INTEGER
The global row index of the submatrix of the distributed matrix X to operate on.
JX (global input) INTEGER
The global column index of the submatrix of the distributed matrix X to operate on.
DESCX (global and local input) INTEGER array of dimension 8
The array descriptor of the distributed matrix X.
INCX (global input) INTEGER
The global increment for the elements of X. Only two values of INCX are supported in this version, namely 1 and M_X. *sub(X)* is an *N* element vector and *sub(A)* is an *N*-by-*N* unit, or non-unit, upper or lower triangular distributed matrix.

Notes and Coding Rules

1) These subroutines accept lowercase letters for the uplo, transa, and diag arguments.
2) For PDTRSV, if you specify 'C' for transa, it is interpreted as though you specified 'T'.
3) The matrix and vector must have no common elements; otherwise, results are unpredictable.
4) PDTRSV and PZTRSV assume certain values in your array for parts of a triangular matrix. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the strictly lower or upper triangular part, respectively, are assumed to be zero. As a result, you do not have to set these values.
5) The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above.
6) The following values must be equal: CTXT_A = CTXT_X.
7) The global triangular matrix *A* must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
8) The block row and block column offsets of the global triangular matrix *A* must be equal; that is, mod(ia-1,MB_A) = mod(ja-1,NB_A).
9) If incx = M_X:
a) The following block sizes must be equal: NB_X = MB_A = NB_A
b) If transa = 'T', then (in the process grid) the process column containing the first column of the submatrix *A* must also contain the first column of the submatrix *X*; that is, iacol = ixcol, where:
i) iacol = mod((((ja-1)/NB_A)+CSRC_A), q)
ii)ixcol = mod((((jx-1)/NB_X)+CSRC_X), q)
c) The block column offset of *x* must be equal to the block row and block column offsets of *A*; that is, mod(jx-1, NB_X) = mod(ja-1, NB_A) = mod(ia-1, MB_A).
10)If incx = 1($\neq$ M_X):
a) The following block sizes must be equal: MB_X = MB_A = NB_A
b) If transa = 'N', then (in the process grid) the process row containing the first row of the submatrix *A* must also contain the first row of the submatrix *X*; that is, iarow = ixrow, where:
i) iarow = mod((((ia-1)/MB_A)+RSRC_A), p)
ii)ixrow = mod((((ix-1)/MB_X)+RSRC_X), p)
c) The block row offset of *x* must be equal to the block row and block column offsets of *A*; that is, mod(ix-1, MB_X) = mod(ia-1, MB_A) = mod(ja-1, NB_A).

Example 3.2 *Utilizarea functiilor pdgemv _ sipdtrsv_*

```
 #include <stdio.h>
 #include <string.h>
 #include <stdlib.h>
 #include "mpi.h"
 #include <iostream>
 #include <iomanip>
 #include <fstream>
  #include <sstream>
 using namespace std;
#define AA(i,j) AA[(i)*M+(j)]
#define AATRI(i,j) AATRI[(i)*M+(j)]
extern "C"
   {
   void Cblacs_pinfo( int* mypnum, int* nprocs);
   void Cblacs_get( int context, int request, int* value);
   int  Cblacs_gridinit( int* context, char * order, int np_row, int np_col);
   void Cblacs_gridinfo( int context, int*  np_row, int* np_col, int*  my_row, int*  my_col);
   void Cblacs_gridexit( int context);
   void Cblacs_barrier(int, const char*);
   void Cblacs_exit( int error_code);
   void Cblacs_pcoord(int, int, int*, int*);
   int  numroc_( int *n, int *nb, int *iproc, int *isrcproc, int *nprocs);
   int indxl2g_(int*, int*, int*, int*, int*);
   void descinit_( int *desc, int *m, int *n, int *mb, int *nb, int *irsrc, int *icsrc, int *ictxt, int
*lld, int *info);
   void pdgemv_(char *trans, int *m, int *n, double *alpha, double *A, int *ia, int *ja, int
*desc_A, double *X, int *ix,
      int *jx, int*desc_X, int *incx, double *beta, double *Y, int *iy, int *jy, int *desc_Y, int
*incy );
   void pdtrsv_(char *UPLO, char *trans, char *DIAG, int *N, double *A, int *IA, int *JA, int
*DESCA, double *X, int *IX,
       int *JX, int *DESCX, int *INCX);
   } // extern "C"
   int main(int argc, char **argv) {
 int i, j, k;
   /************  MPI ***************************/
 int myrank_mpi, nprocs_mpi;
 MPI_Init( &argc, &argv);
 MPI_Comm_rank(MPI_COMM_WORLD, &myrank_mpi);
 MPI_Comm_size(MPI_COMM_WORLD, &nprocs_mpi);
   /************  BLACS ***************************/
 int ictxt, nprow, npcol, myrow, mycol,nb,myid;
 int info,itemp;
 int ZERO=0,ONE=1;
 nprow = 2; npcol = 2; nb =2;
   Cblacs_pinfo( &myrank_mpi, &nprocs_mpi) ;
   Cblacs_get( -1, 0, &ictxt );
   Cblacs_gridinit( &ictxt, "Row", nprow, npcol );
   Cblacs_gridinfo( ictxt, &nprow, &npcol, &myrow, &mycol );
 int M=5;
 double *AA = (double*) malloc(M*M*sizeof(double));// matricea "globala" pentru inmultire
matrice cu vectorAA*X=Y
```

```
  double *AATRI = (double*) malloc(M*M*sizeof(double));//matricea "globala" rezolv.
sitemului de ecuatii AATRI*Y=B
  for(i=0;i<M;i++ )
    for(j=0;j<M;j++)
  AA[i*M+j]=(2*i+3*j+1);
    //Se complecteaza cu valori matricea "triungiulara"
    for(i=0;i<M;i++ )
    for(j=0;j<M;j++)
    {if (j>i) AATRI[i*M+j]=0;
     else       AATRI[i*M+j]=(2*i+3*j+1);
    }
    // Tipar Matricele globale (initiale)
    if (myrank_mpi==0)
    {
    printf("============ REZULT OF THE PROGRAM %s \n",argv[0]);
    cout << "Global matrix AA:\n";
        for (i = 0; i < M; ++i) {
            for (int j = 0; j < M; ++j) {
                cout << setw(3) << *(AA + M*i + j) << " ";
            }
            cout << "\n";
        }
        cout << endl;
    cout << "Global matrix AATRI:\n";
        for (i = 0; i < M; ++i) {
            for (int j = 0; j < M; ++j) {
                cout << setw(3) << *(AATRI + M*i + j) << " ";
            }
            cout << "\n";
        }
        cout << endl;
    }
  double *X = (double*) malloc(M*sizeof(double)); // X- vectorul "global"
  double *XTRI = (double*) malloc(M*sizeof(double)); // XTRI- vectorul "global"(partea
dreapta a siste. de ecuatii)
  X[0]=1;X[1]=1;X[2]=0;X[3]=0;X[4]=1;
  XTRI[0]=1;XTRI[1]=1;XTRI[2]=0;XTRI[3]=0;XTRI[4]=1;
  int descA[9],descAtri[9],descx[9],descxtri[9],descy[9];
 int mA = numroc_( &M, &nb, &myrow, &ZERO, &nprow );
  int nA = numroc_( &M, &nb, &mycol, &ZERO, &npcol );
  int mAtri = numroc_( &M, &nb, &myrow, &ZERO, &nprow );
  int nAtri = numroc_( &M, &nb, &mycol, &ZERO, &npcol );
  int nx = numroc_( &M, &nb, &myrow, &ZERO, &nprow );
  int my = numroc_( &M, &nb, &myrow, &ZERO, &nprow );
  descinit_(descA, &M,  &M,  &nb,  &nb,  &ZERO, &ZERO, &ictxt, &mA,  &info);
  descinit_(descAtri, &M,  &M,  &nb,  &nb,  &ZERO, &ZERO, &ictxt, &mA,  &info);
  descinit_(descx, &M, &ONE,  &nb, &ONE,  &ZERO, &ZERO, &ictxt, &nx, &info);
  descinit_(descxtri, &M, &ONE,  &nb, &ONE,  &ZERO, &ZERO, &ictxt, &nx, &info);
  descinit_(descy, &M, &ONE,  &nb, &ONE,  &ZERO, &ZERO, &ictxt, &my, &info);
  double *x = (double*) malloc(nx*sizeof(double)); //x- vectorul local
  double *xtri = (double*) malloc(nx*sizeof(double)); //xtri- vectorul local
  double *y = (double*) calloc(my,sizeof(double));
```

```
  double *A = (double*) malloc(mA*nA*sizeof(double)); //matricea locala p-ru inlultirea cu
vector
  double *Atri = (double*) malloc(mA*nA*sizeof(double)); //matricea locala p-ru rez. sist. de
ecuatii
  int sat,sut;
    // Se complecteaza cu valor vectorul si matricea locala
    for(i=0;i<mA;i++)
       for(j=0;j<nA;j++){
          sat= (myrow*nb)+i+(i/nb)*nb;
          sut= (mycol*nb)+j+(j/nb)*nb;
          A[j*mA+i]=AA(sat,sut);
          Atri[j*mA+i]=AATRI(sat,sut);
  }
  for(i=0;i<nx;i++){
          sut= (myrow*nb)+i+(i/nb)*nb;
          x[i]=X[sut];
      xtri[i]=X[sut];
  }
  double alpha = 1.0; double beta = 0.0;
    pdgemv_("N",&M,&M,&alpha,A,&ONE,&ONE,descA,x,&ONE,&ONE,descx,&ONE,&beta,
y,&ONE,&ONE,descy,&ONE);
    if (myrank_mpi==0) printf("Final rezult of matrix-vector multiplication \n");
    if (mycol==0)
    {
     for(i=0;i<my;i++)
  printf("For rank=%d y[%d](gl.ind=%d) =%.2f \n", myrank_mpi,i,indxl2g_(&i, &nb, &myrow,
&ZERO, &nprow),y[i]);
    }
    Cblacs_barrier(ictxt, "All");
    pdtrsv_("L", "N", "N", &M, Atri, &ONE, &ONE, descAtri, xtri, &ONE,&ONE,descxtri,
&ONE);
    if (myrank_mpi==0) printf("Final rezult of solves of the systems of equations  \n");
    Cblacs_barrier(ictxt, "All");
    if (mycol==0)
    {
    for(i=0;i<nx;i++)
    printf("For rank=%d x[%d](gl.ind=%d) =%.2f \n", myrank_mpi,i,indxl2g_(&i, &nb,
&myrow, &ZERO, &nprow),xtri[i]);
    }
    Cblacs_barrier(ictxt, "All");
  Cblacs_gridexit(0);
  MPI_Finalize();
  return 0;
    }
```

Result of compilation and executing

```
    [Hancu_B_S@hpc Pentru_Masterat]$ ./mpiCC_ScL -o Example3.2.exe Example3.2.cpp

    [Hancu_B_S@hpc Pentru_Masterat]$ /opt/openmpi/bin/mpirun -n 4 -host compute-0-0
Example3.2.cpp
    ============ REZULT OF THE PROGRAM Use_pblas_L2.exe
    Global matrix AA:
      1   4   7  10  13
```

```
 3   6   9  12  15
 5   8  11  14  17
 7  10  13  16  19
 9  12  15  18  21
Global matrix AATRI:
 1   0   0   0   0
 3   6   0   0   0
 5   8  11   0   0
 7  10  13  16   0
 9  12  15  18  21
Final rezult of matrix-vector multiplication
For rank=0 y[0](gl.ind=0) =18.00
For rank=0 y[1](gl.ind=1) =24.00
For rank=2 y[0](gl.ind=2) =30.00
For rank=2 y[1](gl.ind=3) =36.00
For rank=0 y[2](gl.ind=2) =42.00
Final rezult of solves of the systems of equations
For rank=0 x[0](gl.ind=0) =1.00
For rank=0 x[1](gl.ind=1) =-0.33
For rank=2 x[0](gl.ind=2) =-0.21
For rank=2 x[1](gl.ind=3) =-0.06
For rank=0 x[2](gl.ind=2) =0.01
```

Explicatii suplementare pentru Exemplu 3.2 (vezi fisierul *Pentru Example3.2.docx*)